Crossovers from parity conserving to directed percolation universality

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The crossover behavior of various models exhibiting phase transition to absorbing phase with parity conserving class has been investigated by numerical simulations and cluster mean-field method. In case of models exhibiting Z_2 symmetric absorbing phases (the cellular automaton version of the nonequilibrium kinetic Ising model (NEKIMCA) and a stochastic cellular automaton invented by Grassberger, Krause, and von der Twer [J. Phys. A **17**, L105 (1984)]) the introduction of an external symmetry breaking field causes a crossover to kink parity conserving models characterized by dynamical scaling of the directed percolation (DP) and the crossover exponent: $1/\phi \simeq 0.53(2)$. In the case of a branching and annihilating random walk model with an even number of offspring (dual to NEKIMCA) the introduction of spontaneous particle decay destroys the parity conservation and results in a crossover to the DP class characterized by the crossover exponent: $1/\phi \simeq 0.205(5)$. The two different kinds of crossover operators cannot be mapped onto each other and the resulting models show a diversity within the DP universality class in one dimension. These subclasses differ in cluster scaling exponents.

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I. INTRODUCTION

The study of nonequilibrium phase transitions is an important task of statistical physics. Genuinely nonequilibrium transitions can be observed most easily in models exhibiting transition from an active to an "absorbing" state, where the fluctuations are negligible, hence no return is possible. The exploration of critical phenomena and universality classes of simple, one component models has been started [1–4] and important steps towards a full classification have been done [5,6].

For a long time it was a common belief that all continuous, nonequilibrium phase transitions belong to class of the directed percolation (DP) and a hypothesis advanced by Janssen and Grassberger [7–9]. This states that in one component systems exhibiting continuous phase transitions to a single absorbing state (without extra symmetry and inhomogeneity or disorder) short-ranged interactions can generate DP class transition only. Despite the robustness of this class experimental observation is rare, owing to the high sensitivity to disorder and long-range interactions. A very recent experimental study [10] has reported clear and comprehensive experimental evidence of DP criticality. Later it was discovered that in systems with infinitely many frozen absorbing states (IM type) [11–14] like in the pair contact process (PCP) the static exponents coincide with those of the DP. The dynamical cluster spreading behavior is different owing to the long time memory generated by the frozen monomers [15]. In the literature the transition type of PCP is often called DP type. Very recently [16] have investigated in $1+1$ dimension the crossover from PCP to DP type of models by introducing different absorbing state reduction mechanisms and nontrivial exponents have been found. This finding confirms that strictly speaking the universality class of PCP and DP is different.

The first example for clearly non-DP critical behavior was found among stochastic cellular automata (SCA) by Grassberger [17]. These models exhibit Z_2 symmetric absorbing states and an effective kink dynamics, which follows an even-offspringed branching and annihilating random walk (BARWe). In reaction-diffusion (RD) particle models, with BARWe dynamics ($A \to 3A$, $2A \to \emptyset$) such phase transition class was discovered by [18,19]. Since this is different from odd-offspringed branching and annihilating random walks, in which DP class transition occurs, the name "parity conserving" (PC) was introduced to denote this class.[1]

An important example of the PC class behavior was discovered in one-dimensional kinetic Ising models with combined zero temperature spin-flip and finite temperature spin-exchange dynamics (NEKIM) [20]. Here the domain walls between up and down spins follow BARWe dynamics and an exact duality transformation in one dimension between the NEKIM and the BARWe particle model was established by [21].[2] Naturally the NEKIM exhibits two, Z_2 symmetric absorbing states. This universality class has not been observed in nature yet, however we pointed out [22] that this model is very insensitive to the quenched disorder, therefore it is a good candidate for experimental verification. The inactive phase of NEKIM, where annihilating random walk dominates has been observed [23] unless very strong disorder fractures the medium.

The introduction of a symmetry breaking external magnetic field, in NEKIM, which favors one of the absorbing states (but preserves the BARWe dynamics) was shown to change the type of transition from PC to DP type [24]. Such crossover mechanism has been observed in other Z_2 symmetric models as well [25–29]. On the other hand simulations [30] and field theory [31,32] proved that a PC breaking $A \to \emptyset$ reaction in the BARWe model should also change the type of phase transition from PC to DP type.

The more detailed study of crossover behavior among nonequilibrium universality classes has been intensified in

[1]However, other different names for this class such as "directed Ising" (DI) or "generalized voter (GV)" or "BARW" can also be found in the literature

[2]Note that such mapping is not possible in higher dimensions, but since the upper critical dimension is $d_c=4/3$ it is not important.

the recent years. The universal crossover exponent, defined by the shape of phase boundary $[r_c(w)]$ as one introduces a relevant scaling field (w) (see for example [33]),

$$r_c(w) \sim w^{1/\phi}, \tag{1}$$

has been determined in case of DP to the compact DP class [34–37]. Crossover between DP and isotropic percolation was investigated by field theory [38–40] and simulations [41]. Very recently the numerical exploration of the crossover behavior from the diffusive pair contact process (PCPD) to DP [42] has strengthened the existence of the independent PCPD class theory. In IM type models "nontrivial crossovers to DP class" have been found [16]. The crossover behavior from DP to mean-field, generated by long-range diffusion, following the early studies [43,44], has currently been reexamined via numerical techniques [45] and field theory [46] and estimates for the exponent ϕ have been provided. Similar, diffusion driven crossover in case of PC class was determined within the framework of NEKIM [47] long ago.

In the present paper we determine the crossover exponent from PC to DP class in a SCA version of the NEKIM model [24] (NEKIMCA) and compare it with other realizations of the PC class. In particular we confirm the universality of ϕ in case of Z_2 symmetry breaking fields by simulating Grassberger's A model. We compare this crossover behavior with the outcome of parity conservation breaking in a BARWe model.

II. CROSSOVER OF NEKIMCA IN AN EXTERNAL FIELD

The NEKIM exhibiting PC class transition was suggested by [20] as a generalization of the Glauber Ising model [48]. It is defined by the alternating application of a $T=0$ spin-flip sweep and a $T>0$ Kawasaki spin-exchange update of a one-dimensional lattice. While the spin-flip dynamics generates annihilating random walk of kinks, the spin-exchange introduces a parity conserving branching $(A \to 3)$ of the domain walls. Tuning the relative strengths of the reactions one can get a phase transition from an active to a kink free, adsorbing state (the order parameter is the density of kinks).

It was realized in [24] that branching reactions appear automatically in the SCA version of the NEKIM, if the spin-flip update is done synchronously due to overlaps. With this dynamics we can obtain a very simple model, with PC type of critically, which can be implemented on a computer by efficient bit coding. The NEKIMCA spin updates, represented by bit field operations of a computer word, generating BARWe reactions of the kinks are the following.

Random walk of domain walls (•):

$$\uparrow\uparrow \bullet \downarrow \xrightarrow{w_i} \uparrow \bullet \downarrow\downarrow$$

generated by a spin-flip between oppositely oriented spins, with probability w_i.

Annihilation of a pair of kinks:

$$\uparrow \bullet \downarrow \bullet \uparrow \xrightarrow{w_o} \uparrow\uparrow\uparrow$$

generated by a spin-flip between a pair of same oriented spins, with probability w_o.

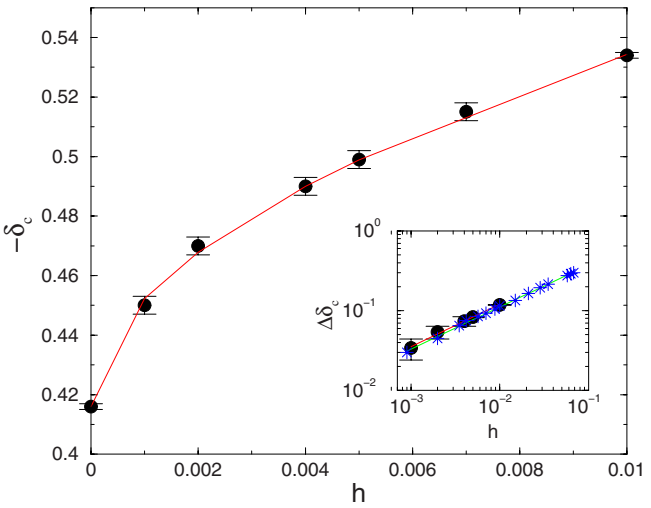

FIG. 1. (Color online) Critical point shift in the NEKIMCA model as a result of the external h field for $\Gamma=2$. The solid line is power-law fitting on the numerical data points. The inset shows the scaling of the shift of $\tilde{\delta}$ for $\Gamma=2$ (bullets) and for $\Gamma=0.35$ (stars) on a log-log scale.

Branching of a kink:

$$\uparrow\uparrow \bullet \downarrow\downarrow \xrightarrow{w_i^2} \uparrow \bullet \downarrow \bullet \uparrow \bullet \downarrow$$

can be generated by two overlapping spin-flips (with the probability w_i^2) such that it causes a spin-exchange.

There are two independent parameters related to the original parametrization of Glauber,

$$w_i = \Gamma(1-\tilde{\delta})/2, \tag{2}$$

$$w_o = \Gamma(1+\tilde{\delta}). \tag{3}$$

In the present study we investigated the $\Gamma=0.35$ and $\Gamma=2$ cases. The corresponding PC critical points, without external field, are located at $\tilde{\delta}_c=-0.535$ and $\tilde{\delta}_c=-0.416$, respectively.

A single Monte Carlo step (MCS) consists of updating all sites at once as described in [49] (throughout the paper the time is measured by MCS). The simulations were reformed on $L=40000$ sized lattices, with periodic boundary conditions up to $t_{\max}=10^6$ MCS. The change of critical point $\delta_c(h)$ has been determined for several values of external field. The transition probabilities are modified in the presence of an external magnetic field H as

$$w_i^h = w_i(1-hs_i), \tag{4}$$

$$w_o^h = w_o(1-hs_i), \tag{5}$$

$$h = th\left(\frac{H}{kT}\right). \tag{6}$$

Figure 1 shows the phase diagram in the $(h,\tilde{\delta})$ plane, which is similar to the one we obtained in [24] by simulations and cluster mean-field technique. We have applied only random initial state simulations to find the points of the line of phase

transition exhibiting kink density decay exponent $\alpha = 0.1595$ of the DP [3]. A clear power-law scaling of the critical point shift $\Delta\tilde{\delta}$ as the function of h can be fitted with the form (1) in the region $0<h<0.01$ with $1/\phi=0.52(3)$ crossover exponent. The error bars on Fig. 1 come from our numerical estimates for the critical point shifts and the least squares error estimate fitting procedure. This value is close to the early numerical estimates: $\phi=2.1(1)$ by [26] and $\phi=2.24(10)$ by [29].

III. CROSSOVER OF THE GRASSBERGER-A SCA MODEL

The PC conserving SCA by Grassberger is realized by the following range-1 update rule (we show the configurations at $t-1$ and the probability p of getting "1" at time t):

t−1:	100	001	101	110	011	111	000	010
t:	1	1	0	1−p	1−p	0	0	1

The time evolution pattern in 1+1 dimension, for small p evolves towards a stripelike ordered steady state (with double degeneration), while for $p>p_c=0.1245(5)$ the kinks (the "00" and "11" pairs) survive. According to the classification of Wolfram [50] for $p=0$ we have the Rule-94, class 1 CA, while the $p=1$ limit is the chaotic Rule-22 deterministic CA. This model has been investigated from damage spreading point of view by the present authors [51].

Now we extend this model in such a way that a Z_2 symmetry breaking occurs in the absorbing state. This can be achieved by favoring one of the absorbing phases ("1010101" or "0101010") shifted by a single site. Therefore we modified the transition rates

t−1:	100	001	101	110	011	111	000	010
t:	1	1	0	1−p′	1−p′	0	0	1

such that $p'=p+w$ at odd and $p'=p-w$ at even sites. The simulations were run on $L=10^5$ sized rings up to $t_{\max}=10^6$ MCS. The location of the critical point $p(w)$ is shown on Fig. 2. To get more precise estimates for the crossover exponent we determined the local slopes of the data points,

$$1/\phi_{\text{eff}}(w_i) = \frac{\ln r_c(w_i) - \ln r_c(w_{i-1})}{\ln(w_i) - \ln(w_{i-1})}. \tag{7}$$

This is plotted in the inset of Fig. 2 and in the $w\to 0$ limit one can read-off the $1/\phi=0.53(2)$ linear extrapolation value (with least squares error estimates) in agreement with the value for NEKIMCA+h. As the figure shows the correction to scaling cannot be neglected.

IV. CROSSOVER IN A BARWE MODEL

To study the PC to DP crossover in another way we take a simple version of a BARWe particle model introduced in [52] and generalized in [53] (ZAMb model) to allow phase transition at finite branching rate. This model is defined on the one-dimensional lattice as follows. An occupied site is chosen randomly and is tried for diffusion, with probability D, or branching with probability $\sigma=1-D$; while the time is increased by $1/N$, where N is the number of occupied (active) sites. In a diffusion step the particle jumps to its randomly chosen nearest-neighbor sites. If the site is occupied both particles are annihilated with probability r. On the other hand, the jump is rejected with probability $1-r$. The branching process involves the creation of two new particles around the neighborhood. If either, or both neighboring sites is previously occupied the target site(s) become empty with probability r. Otherwise, the lattice remains unaltered with probability $1-r$.

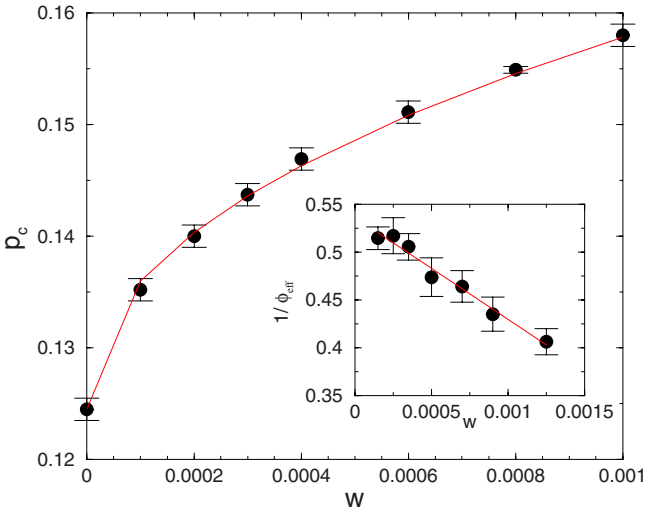

FIG. 2. (Color online) Critical point shift of the Grassberger's-A SCA model to DP (bullets). The lines show power-law-type fitting on the numerical data. The inset shows the local slopes of the crossover exponent defined as (7).

To induce a crossover to DP we introduced a spontaneous particle removal $A\to\emptyset$ or a coagulation $AA\to A$ with a small probability w to the above reactions. The simulations were done on lattices of size $L=10^5$ with periodic boundary conditions for diffusion rate $D=0.2$ (the convergence of GMF approximations was found to be rather good for this diffusion rate in [53]). For $w=0$ we used the critical point value determined in [53]: $r_c=0.562(1)$. As we increase w the critical point $r_c(D)$ shifts as shown in Fig. 3. A simple power-law fitting for the data in the range $w\in(0,0.1)$ of the form 1 resulted in $\Delta r_c=0.189w^{0.181}$. A more precise estimate, which takes the corrections to scaling into account can be obtained by calculating the local slopes (7). By plotting the effective exponent as the function of w a linear extrapolation resulted in $1/\phi=0.205(5)$ both for the $AA\to A$ and the $A\to\emptyset$ parity breaking reactions.[3]

V. GMF+CAM CALCULATIONS

The simulation results were complemented by analytical cluster mean-field approximations and coherent anomaly ex-

[3]Note that the sign of correction to scaling is different for the two crossover versions.

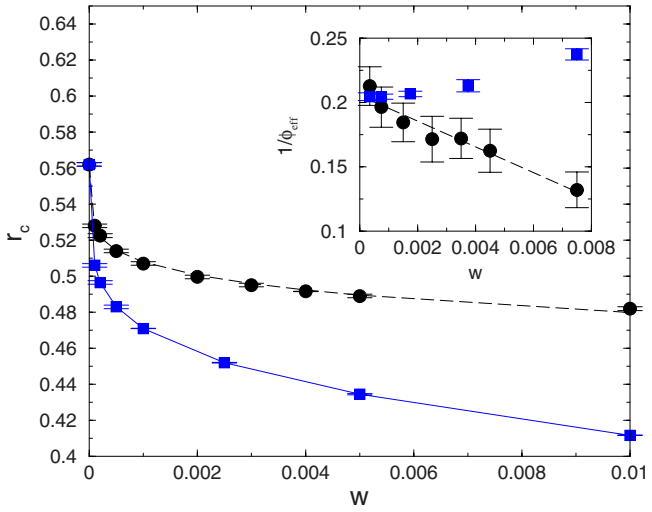

FIG. 3. (Color online) Critical point shift in the ZAMb model to DP as the result of $AA \to A$ (bullets) and $A \to \emptyset$ (boxes) parity conservation breaking. Dashed lines are power-law-type fitting to numerical data. The inset shows the corresponding effective exponents defined as (7).

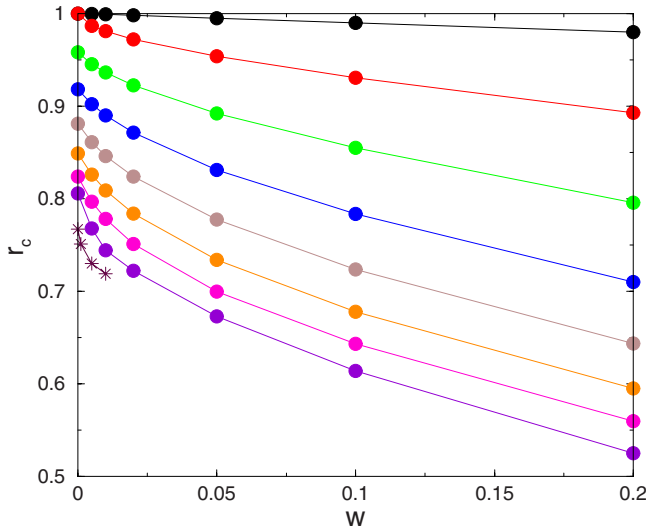

FIG. 4. (Color online) Critical point shift of the ZAMb model to DP (bullets) obtained by the GMF method for various cluster sizes ($N=1,2,\ldots,9$ top to bottom).

trapolations. The generalized (cluster) mean-field method (GMF) suggested for SCA by [54] and for dynamical RD models by [55] has been shown to be a successful approach for exploring the phase diagram of nonequilibrium models (see, for example, [56,57] and the references in [3]). It's extension with the coherent anomaly (CAM) extrapolation [58] enables one to extract the true scaling behavior.

In GMF we set up equations for the steady state of the system based on N-point block probabilities. Correlations with a range greater than N are neglected. By increasing N from 1 (traditional mean-field) step by step we take into account more and more correlations and get better approximations. One can set up master equations for the P_N block probabilities as

$$\frac{\partial P_N(\{s_i\})}{\partial t} = f(P_N(\{s_i\})), \tag{8}$$

where the site variables take the values $s_i = \emptyset, A$. During the solution of these equations one estimates larger than N sized block probabilities by the maximum overlap approximation:

$$P_{N+1}(s_1, \ldots, s_{N+1}) \simeq \frac{P_N(s_1, \ldots, s_N) P_N(s_2, \ldots, s_{N+1})}{P_N(s_2, \ldots, s_N, \emptyset) + P_N(s_2, \ldots, s_N, A)}. \tag{9}$$

Taking into account the spatial symmetries and the conservation of probability for the maximal, $N=9$ approximation of this work we had to find the solution of a set of nonlinear equations of 272 independent variables.

The steady-state solutions of the N-cluster approximations for the NEKIMCA ($1 \leqslant N \leqslant 8$) and the ZAMb model ($1 \leqslant N \leqslant 9$) have been determined and the corresponding densities are calculated numerically. The phase transition points are obtained for several values of the crossover parameter in both cases. In case of the ZAMb model (at $D=0.2$) they are plotted in Fig. 4. As one can see for $N=1$ (site mean-field) the critical point $r_c(N,w)$ exhibits linear relation without corrections to scaling. This also true for the ZAMB+sink case. This means the $1/\phi_{MF}=1$ mean-field value for both kinds of crossover.

For $N>1$ the $r_c(N,w)$ curves pick up $w^{1/2}$ type of corrections to scaling (the leading order singularity is expected to remain mean-field like). Assuming a scaling with the correction form

$$r_c(N,w) = a(N)w + b(N)w^{1/2}, \tag{10}$$

which is natural if we swap the axes r_c and w, one can determine the amplitudes $[a(N)]$ of the leading order term. Following the envelope scaling hypothesis of [58] the coherent anomaly method (CAM) extrapolation can be performed on the amplitude data in the $N \to \infty$ limit.

According to CAM the amplitudes $a(N)$ of the cluster mean-field singularities in the leading order scale as

$$|a(N)| \propto |r_c(N) - r_c|^{1/\phi - 1/\phi_{\mathrm{MF}}} \tag{11}$$

allowing us to estimate the $1/\phi$ exponent of the true singular behavior [Eq. (1)]. The $a(N)$ amplitudes were determined by a next leading order fitting form (10) on the $r_c(N,w)$ data in the neighborhood of $r_c(N,0)$ for the ZAMb model. In case of the NEKIMCA the same procedure has been applied for the $\tilde{\delta}_c(N,h)$ crossover critical point GMF results.

The highest $a(N)$ amplitudes with a CAM fitting form (which takes into account possible scaling corrections [56])

$$a(N) = c\Delta_c^x + d\Delta_c^{x+1}, \tag{12}$$

where $\Delta_c(N) = |\tilde{\delta}_c - \tilde{\delta}_c(N)|$ for NEKIMCA and $\Delta_c(N) = |r_c - r_c(N)|$ for the ZAMb are plotted on Fig. 5. In this form the nonuniversal fitting parameters are c, d and the anomaly exponent is $x = 1/\phi - 1/\phi_{\mathrm{MF}}$. By the simulations (apart from the constants c and d) we have an expected behavior for (12) both for ZAMb and NEKIMCA. This is plotted on Fig. 5 by the dashed lines. As one can see our GMF data can be fitted

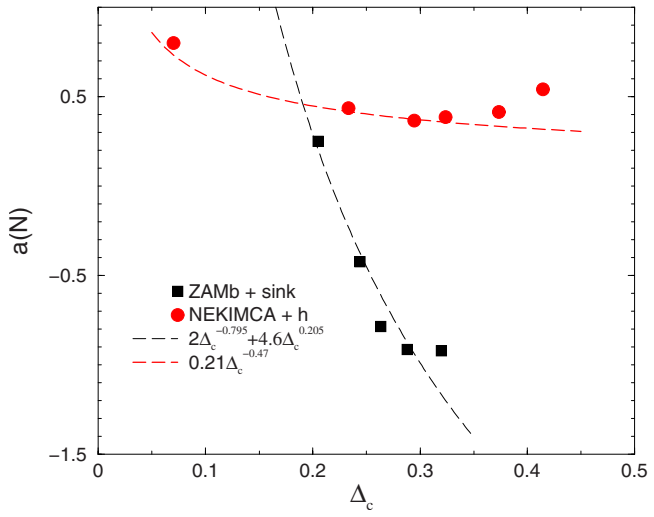

FIG. 5. (Color online) CAM extrapolation results for the anomaly amplitudes $a(N)$. Bullets, NEKIMCA+h ($3 \leqslant N \leqslant 8$ right to left), boxes, ZAMb+sink ($5 \leqslant N \leqslant 9$ right to left). The numerical errors are smaller than the symbol sizes. The dashed lines show the expected critical behavior from simulations.

with those lines (with the values $c=2$, $d=4.6$ for ZAMb and $c=0.21$, $d=0$ for NEKIMCA) but the amplitudes for $N<10$ just start to converge towards the asymptotic scaling curves. This suggests that larger N-cluster approximations should be determined. However, the numerical instability of root finding of the GMF method (within the space of more than 500 variable, nonlinear system) prevented us to go further. Still the different behavior for NEKIMCA and ZAMb can be justified by this figure.

VI. CONCLUSIONS AND DISCUSSION

We have performed simulations and GMF+CAM approximations for various versions of the PC class model crossovers towards the "DP class." By determining the crossover exponents we have found and outstanding difference between the effect of a symmetry breaking field $[1/\phi=0.53(2)]$ and the parity conservation breaking $[1/\phi=0.205(5)]$. Although the NEKIMCA and the BARWe are dual models, the two types of crossover operators cannot be mapped onto each other. Furthermore they cannot be mapped onto a local one in the dual system. The $A \to \emptyset$ process in the spin language would mean flipping of all spins from a given site (see Fig. 6). Such transformation cannot even be done in finite systems with periodic boundary conditions.

A Z_2 symmetry breaking in the BARWe model favors the "+" (up) or "−" (down) oriented spins depending on its sign. In the case of a "+" preference (see Fig. 6) this means that "+" domains broaden on the expense of "−" domains, hence odd-even kinks (the corresponding particle pairs in BARWe) attract and even-odd pairs repel each other (shown by the horizontal arrows). Therefore, that particle model becomes an effectively two-component, parity conserving one, and the sufficient conditions of the DP hypothesis do not hold. Still one can see a DP type of decay in the global order parameter because calling a "+" domain a macroscopic particle "X," an effective DP process $X \to \emptyset$, $X \to 2X$, $2X \to X$ describes it's dynamics.

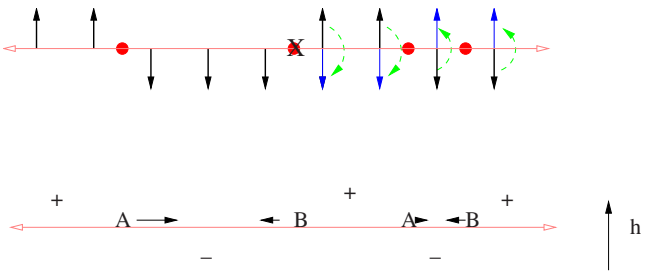

FIG. 6. (Color online) Schematic mapping of the two different kinds of crossovers between the kink and the spin model. The top graph shows that the removal of a single kink corresponds to flipping of all spins to the right from position X. The bottom graph demonstrates that an external magnetic field in the spin model causes repulsion-attraction force is among domain walls (horizontal arrows) and an effectively two-component reaction-diffusion system of kinks (particles) emerges.

However, there are certain operators which do not exhibit DP type behavior. For example, in case of cluster spreading it is easy to see that there are two sectors in this model. An odd parity one, with $\beta'=0$ final survival probability defined as

$$P_\infty \propto |p-p_c|^{\beta'} \tag{13}$$

and an even parity one, with normal, $\beta'=\beta$ DP exponent. As a consequence the hyperscaling relation connecting the cluster spreading exponents and the rapidity reversal symmetry of DP is not satisfied for this critical behavior. The universality class of the DP is split into subclasses. The subclass of models with BARWe dynamics with broken Z_2 symmetry (let us call it DP-2) is different from that of the ordinary (1+1)-dimensional DP. Since the difference is manifested in the cluster spreading behavior, which is a consequence of a special initial condition, similarly to the terminology used in equilibrium models with different surface classes we do not claim the splitting of the DP class itself.

This subclass behavior is similar to the one, which is observed in the pair contact process, where the frozen monomers cause different cluster behavior from that of the simple contact process. Also that kind of DP-2 behavior should arise, when one introduces spin-anisotropy in the NEKIM [59,60] and the effective two-component BARWe model exhibits DP type of phase transition for finite branching rate in the global order parameter, but different scaling behavior occurs in terms the cluster spreading. Furthermore this model exhibits a reentrant phase diagram and at zero branching rate one finds another phase transition belonging to the two-component BARWe model [31], but the cluster spreading behavior, which is sensitive to the spin anisotropy is different again. Therefore such subclasses are not at all rare among low-dimensional reaction-diffusion models.

We would like to point out that by using an alternative definition for universality classes a different interpretation can also be given for the same numerical results presented here. Some authors define a university class by the field-

theoretic action of the model (without specifying the fixed point) instead of the models exhibiting the same set of known critical exponents as we do. Since the action of BARWe particles [31] and the action of the NEKIM spin model [32] (called GV model) do not agree, they should belong to different universality classes, which "intersect" in one dimension only [2]. According to this picture the applied symmetry breaking to the PC and GV class models (Z_2 breaking in GV and particle removal in BARWe) should end up in DP class somehow.

However we think that our definition of universality class, and therefore our interpretation, is more precise. It can describe the critical behavior of (particle) system for which no proper field theoretical action has been found. Even within a field theory multiple fixed point solutions (corresponding to multiple critical points) can exist and the relevancy of terms affecting the stability of a fixed point is not clear in many cases.

Furthermore a coarse-grained field theory may not capture all scaling details of a particle model. In a discrete particle model diffusive annihilating particles can die out within finite time (even in an infinite sized system) due to the recurrence relation of random walks in one dimension.[4] The particle survival probability decays asymptotically as $P_s(t) \propto t^{-\delta}$ and the final survival probability scales as (13). As a consequence a thermodynamic limit, in which the particle density is kept finite cannot be established.

On the other hand, in field theories of continuous variables the survival probability is always unity and it had been unclear if the exponent δ and β' could be defined sensibly. To overcome this discrepancy a reinterpretation of survival probability was proposed in [61]. This alternative definition of δ in field theory resulted in correct scaling exponents and hyper scaling relations corresponding to symmetries.

Since in our case the "thermodynamic limit" can not be restricted to infinite particle number the cluster spreading behavior starting from finite number of particles is relevant from the universality point of view as in many papers investigating absorbing phase transition via this approach (see [1–4,11,12,62]. The cluster spreading behavior (the finite survival probability, pair-connectedness functions, avalanche distributions, etc.) are sensitive to the initial parity of particles, hence strictly speaking the DP-2 subclass is not identical to the DP class in 1+1 dimension.

[4]Note that this is not a finite size effect.

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[1] J. Marro and R. Dickman, *Nonequilibrium Phase Transitions in Lattice Models* (Cambridge University Press, Cambridge, 1999).
[2] H. Hinrichsen, Adv. Phys. **49**, 815 (2000).
[3] G. Ódor, Rev. Mod. Phys. **76**, 663 (2004).
[4] S. Lübeck, Int. J. Mod. Phys. B **18**, 3977 (2004).
[5] V. Elgart and A. Kamenev, Phys. Rev. E **74**, 041101 (2006).
[6] G. Ódor, *Universality In Nonequilibrium Lattice Systems* (World Scientific, Singapore, 2008).
[7] H. K. Janssen, Z. Phys. B: Condens. Matter **42**, 151 (1981).
[8] P. Grassberger, Z. Phys. B: Condens. Matter **47**, 365 (1982).
[9] G. Grinstein, Z.-W. Lai, and D. A. Browne, Phys. Rev. A **40**, 4820 (1989).
[10] K. A. Takeuchi, M. Kuroda, H. Chaté, and M. Sano, Phys. Rev. Lett. **99**, 234503 (2007).
[11] I. Jensen and R. Dickman, Phys. Rev. E **48**, 1710 (1993).
[12] I. Jensen, Phys. Rev. Lett. **70**, 1465 (1993).
[13] I. Jensen and R. Dickman, Phys. Rev. E **48**, 1710 (1993).
[14] J. F. F. Mendes, R. Dickman, M. Henkel, and M. C. Marques, J. Phys. A **27**, 3019 (1994).
[15] G. Ódor, J. F. Mendes, M. A. Santos, and M. C. Marques, Phys. Rev. E **58**, 7020 (1998).
[16] S. C. Park and H. Park, Phys. Rev. E **76**, 051123 (2007).
[17] P. Grassberger, F. Krause, and T. von der Twer, J. Phys. A **17**, L105 (1984).
[18] H. Takayasu and A. Y. Tretyakov, Phys. Rev. Lett. **68**, 3060 (1992).
[19] I. Jensen, Phys. Rev. E **50**, 3623 (1994).
[20] N. Menyhárd, J. Phys. A **27**, 6139 (1994).
[21] K. Mussawisade, J. E. Santos, and G. M. Schütz, J. Phys. A **31**, 4381 (1998).
[22] G. Ódor and N. Menyhárd, Phys. Rev. E **73**, 036130 (2006).
[23] R. Kroon, H. Fleurent, and R. Sprik, Phys. Rev. E **47**, 2462 (1993).
[24] N. Menyhárd and G. Ódor, J. Phys. A **29**, 7739 (1996).
[25] H. Park and H. Park, Physica A **221**, 97 (1995).
[26] K. E. Bassler and D. A. Browne, Phys. Rev. Lett. **77**, 4094 (1996).
[27] H. Hinrichsen, Phys. Rev. E **55**, 219 (1997).
[28] W. M. Hwang, S. Kwon, H. Park, and H. Park, Phys. Rev. E **57**, 6438 (1998).
[29] S. Kwon, W. M. Hwang, and H. Park, Phys. Rev. E **59**, 4949 (1999).
[30] S. Kwon and H. Park, Phys. Rev. E **52**, 5955 (1995).
[31] J. L. Cardy and U. C. Täuber, Phys. Rev. Lett. **77**, 4780 (1996).
[32] L. Canet, H. Chate, B. Delamotte, I. Dornic, and M. A. Muñoz, Phys. Rev. Lett. **95**, 100601 (2005).
[33] I. Lawrie and S. Sarbach, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. Lebowitz (Academic, London, 1984), Vol. 9.
[34] J. F. F. Mendes, R. Dickman, and H. Herrmann, Phys. Rev. E **54**, R3071 (1996).
[35] H. K. Janssen, J. Phys.: Condens. Matter **17**, S1973 (2005).
[36] S. Lübeck, J. Stat. Mech.: Theory Exp. (2006), P09009.
[37] W. G. Dantas, M. J. de Oliveira, and J. F. Stilck, Braz. J. Phys.

38, 94 (2008).
[38] E. Frey, U. C. Täuber, and F. Schwabl, Phys. Rev. E **49**, 5058 (1994).
[39] P. Fröjdh and M. den Nijs, Phys. Rev. Lett. **78**, 1850 (1997).
[40] H. K. Janssen and O. Stenull, Phys. Rev. E **62**, 3173 (2000).
[41] N. Inui, H. Kakuno, A. Y. Tretyakov, G. Komatsu, and K. Kameoka, Phys. Rev. E **59**, 6513 (1999).
[42] S. C. Park and H. Park, Phys. Rev. E **73**, 025105(R) (2006).
[43] G. Ódor, N. Boccara, and G. Szabó, Phys. Rev. E **48**, 3168 (1993).
[44] G. Szabó and G. Ódor, Phys. Rev. E **49**, 2764 (1994).
[45] W. G. Dantas, M. J. de Oliveira, and J. F. Stilck, J. Stat. Mech.: Theory Exp. (2007), P08009.
[46] A. Messer and H. Hinrichsen, J. Stat. Mech.: Theory Exp. (2008), P04024.
[47] N. Menyhárd and G. Ódor, J. Phys. A **28**, 4505 (1995).
[48] R. J. Glauber, J. Math. Phys. **4**, 191 (1963).
[49] N. Menyhárd and G. Ódor, Braz. J. Phys. **30**, 113 (2000).
[50] S. Wolfram, Rev. Mod. Phys. **55**, 601 (1983).
[51] G. Ódor and N. Menyhárd, Phys. Rev. E **57**, 5168 (1998).
[52] D. Zhong, D. Avraham, and M. A. Muñoz, Eur. Phys. J. B **35**, 505 (2003).
[53] G. Ódor and A. Szolnoki, Phys. Rev. E **53**, 2231 (1996).
[54] H. Gutowitz, J. Victor, and B. Knight, Physica D **28**, 18 (1987).
[55] R. Dickman, Phys. Rev. A **38**, 2588 (1988).
[56] G. Ódor and A. Szolnoki, Phys. Rev. E **71**, 066128 (2005).
[57] G. Ódor, Phys. Rev. E **70**, 066122 (2004).
[58] M. Suzuki, J. Phys. Soc. Jpn. **55**, 4205 (1986).
[59] N. Menyhárd and G. Ódor, Phys. Rev. E **66**, 016127 (2002).
[60] N. Menyhárd and G. Ódor, Phys. Rev. E **68**, 056106 (2003).
[61] M. A. Muñoz, G. Grinstein, and Y. Tu, Phys. Rev. E **56**, 5101 (1997).
[62] P. Grassberger and A. de la Torre, Ann. Phys. **122**, 373 (1979).